Reshetnyak, Yu. G.
Space mappings with bounded distortion. (English) Zbl 0667.30018

The monograph is an enlarged translation of the original Russian edition called “kniga belaja”, the white book, by the Soviet School of quasiconformal mappings; the name refers to the color of its covers. Reshetnyak has written a series of books bound in covers with different colors. This is his first book translated to English.

The book is devoted to mappings with bounded distortion (BD-mappings for short); another name, widely used nowadays, is quasiregular mappings.

The theory of BD-maps has turned out to form the right generalization of the geometric part of the theory of one complex variable analytic functions to real n-dimensional space. These maps can be described as quasiconformal maps without the homeomorphism requirement.

The author started in 1966 a systematic study of maps with bounded distortion in a series of articles. One of his main results is that a nonconstant BD-map $f$ is discrete (i.e. $f^{-1}(y)$ consists of isolated points) and open. These results are contained in the main chapter II of the book. The chapter also contains basis facts about conformal capacity, local analytic structure of these mappings and normal family properties.

The author mostly uses methods from nonlinear potential theory but the book also develops the main bulk of a more geometric view of the subject (by the Finnish group Rickman, Väisälä and the reviewer) based on modulus and capacity methods.

Chapter I is a preliminary chapter. It contains some real analytic methods (Sobolev spaces etc.), basic facts about Möbius transformations and the analytic definition for BD-maps with examples. The last chapter, Chapter III, is an appendix where auxiliary results on quasilinear elliptic equations, lower semicontinuity results of variational integrals and differentiability properties of functions are presented.

Compared to the Russian original edition there are several additional sections: BD-mappings on Riemannian manifolds, bi-lipschitz mappings, stability of quasi-isometric and BD-mappings on various domains. Most of these results are relatively new and due to Reshetnyak himself or to his students. Only a couple of books dealing with BD-mappings in English have appeared before Reshetnyak’s book. The monograph by O. Lehto and K. I. Virtanen [“Quasiconformal mappings in the plane” (1973; Zbl 0267.30016)] contains a chapter on non-homeomorphic plane BD- mappings. In M. Vuorinen’s book [“Conformal geometry and quasiregular mappings” (1988; Zbl 0646.30025)] mainly distortion boundary behavior of BD-mappings is studied. Clearly Reshetnyak’s book sets a new standard and will be a basic reference book in the field for years to come.

Reviewer: O. Martio

MSC:
30C62 Quasiconformal mappings in the complex plane
30-02 Research exposition (monographs, survey articles) pertaining to functions of a complex variable

Keywords:
non-homeomorphic quasiconformal mappings; quasiregular mappings