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A note of F-topologies. (English) Zbl 0712.54001  

Let $X$ be a set and let $2^X$ denote its power set. A mapping $u : 2^X \to 2^X$ is called an F-topology on $X$ if 1) $u\phi = \phi$; 2) $A \subseteq uA$; 3) $A \subseteq B \Rightarrow uA \subseteq uB$; and 4) $u(uA) = uA$. Recall that any transitive binary relation on a set $S$ is called a quasi-order on $S$. We denote by $A(X)$ the set of all quasi-orders $\rho$ on $2^X$ satisfying the additional conditions: i) $B \subseteq A \Rightarrow A \rho B$; ii) $\phi \rho A \Rightarrow A = \phi$; and iii) if $A \in 2^X$ and $(B_i)_{i \in I}$ is a family in $2^X$ such that $A \rho B_i$ for all $i \in I$, then $A \rho \bigcup_{i \in I} B_i$. Now the main result of the paper under review can be stated as follows: Theorem. Let $\mathcal{B}$ be a cover of $X$ and let $u$ be an F-topology on $X$. Then $\mathcal{B}$ is an open base of $u$ if and only if, for each pair of sets $A, B \in 2^X$, there holds

$$B \subseteq uA \Leftrightarrow (\forall C)(C \in \mathcal{B} \text{ and } A \subseteq X \setminus C \Leftrightarrow B \subseteq X \setminus C).$$

Corollary. Let $\rho$ be a binary relation on $2^X$. Then $u \in A(X)$ if and only if there exists a cover $\mathcal{B}$ of $X$ such that, for each pair of sets $A, B \in 2^X$, there holds

$$A \rho B \Leftrightarrow (\forall C)(C \in \mathcal{B} \text{ and } A \subseteq X \setminus C \Rightarrow B \subseteq X \setminus C).$$

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References:

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