Let $\phi : X \to Y$ be a proper algebraic map with connected fibres from a connected quasi-projective $n$-dimensional complex manifold $X$, $n \geq 2$, onto a quasi-projective variety $Y$ and let $L$ be an algebraic line bundle on $X$, which is very ample relatively to $\phi$. The authors use Reider’s technique [I. Reider, Ann. Math., II. Ser. 127, No.2, 309-316 (1988; Zbl 0663.14010)] in a local setting to provide results about the adjoint bundle $K_X \otimes L^{n-1}$, which generalize those obtained by A. J. Sommese and A. Van de Ven [Math. Ann. 278, 593-603 (1987; Zbl 0655.14001)] in the absolute case, i.e. when $Y$ is a point. The authors prove that, unless $\phi$ exhibits $(X,L)$ as a scroll over a smooth curve, the natural morphism $\phi^* \phi_*(K_X \otimes L^{n-1}) \to K_X \otimes L^{n-1}$ is onto. This allows them to construct a normal quasi-projective space $X'$ and algebraic morphisms with connected fibres $\Phi : X \to X'$, $\phi' : X' \to Y$ such that $\phi = \phi' \circ \Phi$ and $K_X \otimes L^{n-1} = \Phi^* \mathcal{L}$, where $\mathcal{L}$ is a line bundle on $X'$, which is ample and spanned relatively to $\phi'$. If $\dim(X') < \dim(X)$ then there is a precise description of $\phi$, while if $\dim(X') = \dim(X)$ then $\Phi$ defines a sort of relative reduction $(X', L')$ of $(X,L)$, up to which, the authors prove that $K_X \otimes L^{n-1}$ is very ample relatively to $\phi$.

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MSC:
14C20 Divisors, linear systems, invertible sheaves
14F05 Sheaves, derived categories of sheaves, etc. (MSC2010)

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