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A removable singularity condition for maps in $R^n$. (English) Zbl 0719.30012

In this brief article, the author uses the notation $D^+ f(x)$ for

$$\limsup_{y \to x} \frac{\|f(y) - f(x)\|}{\|y - x\|}.$$

With $D$ a domain in $R^n$, the idea is to study those $f : D \to R^n$ having $D^+ f(x) < \infty$ apart from points $x$ lying in a certain exceptional set $K$, assumed to have $m_{n-1}(K) = 0$. However, he does assume that $D^+ f$, as restricted to $D \setminus K$, is locally bounded near points of $K$. Indeed, in his main theorem, where he also assumes that $f$ is continuous, he proves without difficulty that the local bound for $D^+ f$ (on $D \setminus K$) is a local Lipschitz constant, and therefore a local bound for $D^+ f$ (on $D$).

The reviewer feels that the corollaries are somewhat overstated. Denoting the branch set as usual by $B_f$, the conclusion of Corollary 2 that the Hausdorff dimension of $B_f$ exceeds $n-2$ (implicitly if $B_f \neq \emptyset$), seems to overlook the standard double-twist example expressed using cylindrical coordinates in $R^3$ by $(r, \theta, z) \to (r, 2\theta, z)$. For this particular $f$ one has $D^+ f \leq 2$ without exception, yet $B_f$ is the entire $z$-axis. The argument associated to Corollary 2, which the above example does not refute, is basically only that $B_f = \emptyset$ if $m_{n-2}(B_f) = 0$.

MSC:

30C65 Quasiconformal mappings in $R^n$, other generalizations