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It is shown that the ideal boundary of a Hadamard manifold $M$ carries a natural quasiconformal structure if the curvature of $M$ is negatively pinched or $M$ has a cocompact group of isometries. For any topological space $X$ carrying a quasiconformal structure $\beta$, a quasiconformal invariant that generalizes the notion of modulus of a curve family is introduced and used to define the conformal dimension of $(X, \beta)$. This dimension increases under quasiconformal imbedding; for example, a quasisymmetric imbedding between Hadamard manifolds of pinched negative curvature extends to a quasiconformal imbedding between their ideal boundaries. Calculations on the conformal dimension at infinity yield a lower bound for the pinching of negatively curved Riemannian metrics carried by compact quotients of rank one symmetric spaces, and a sharp lower bound for the Hausdorff dimension of the limit set of certain quasiconformal groups.

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MSC:
53C20 Global Riemannian geometry, including pinching
30C65 Quasiconformal mappings in $\mathbb{R}^n$, other generalizations

Keywords:
ideal boundary; Hadamard manifold; quasiconformal structure; quasiconformal imbedding; conformal dimension; Hausdorff dimension

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