Summary: Let us call a (para)topological group strongly submetrizable if it admits a coarser separable metrizable (para)topological group topology. We present a characterization of simply $sm$-factorizable (para)topological groups by means of continuous real-valued functions. We show that a (para)topological group $G$ is a simply $sm$-factorizable if and only if for each continuous function $f : G \to \mathbb{R}$, one can find a continuous homomorphism $\varphi$ of $G$ onto a strongly submetrizable (para)topological group $H$ and a continuous function $g : H \to \mathbb{R}$ such that $f = g \circ \varphi$. This characterization is applied for the study of completions of simply $sm$-factorizable topological groups. We prove that the equalities $\mu G = \omega G = \upsilon G$ hold for each Hausdorff simply $sm$-factorizable topological group $G$, where $\upsilon G$ and $\mu G$ are the realcompactification and Dieudonné completion of $G$, respectively. This result gives a positive answer to a question posed by Arhangel’ski and Tkachenko in 2018. It is also proved that $\upsilon G$ and $\mu G$ coincide for every regular simply $sm$-factorizable paratopological group $G$ and that $\upsilon G$ admits the natural structure of paratopological group containing $G$ as a dense subgroup and, furthermore, $\upsilon G$ is simply $sm$-factorizable. Some results in [Completions of paratopological groups, Monatsh. Math. 183, 699-721 (2017)] are improved or generalized.

MSC:
22A05 Structure of general topological groups
22A30 Other topological algebraic systems and their representations
54H11 Topological groups (topological aspects)
54A25 Cardinality properties (cardinal functions and inequalities, discrete subsets)
54C30 Real-valued functions in general topology

Keywords:
simply $sm$-factorizable; realcompactification; Dieudonné completion; Lindelöf $\Sigma$-space; $\mathbb{R}$-factorizable group

Full Text: DOI

References: