Antolín-Camarena, Omar; Villarreal, Bernardo
Nilpotent n-tuples in SU(2). (English) [Zbl 07298493]

When G is a topological group and Γ is a finitely generated group, the space of group homomorphisms \( \text{Hom}(\Gamma, G) \) can be regarded as a topological subspace of \( G^\Gamma \). This paper considers \( \text{Hom}(\Gamma, G) \) when \( \Gamma \) is a finitely generated nilpotent group and \( G = SU(2) \). The case when \( \Gamma = \mathbb{Z}^n \) has been studied by multiple authors, for instance in [A. Adem and F. R. Cohen, Math. Ann. 338, No. 3, 587–626 (2007; Zbl 1131.57003); T. Baird et al., Ill. J. Math. 55, No. 3, 805–813 (2011; Zbl 1278.55027); D. Kishimoto and M. Takeda, Adv. Math. 386, Article ID 107809, 43 p. (2021; Zbl 07367645)]. The authors identify the connected components of \( \text{Hom}(\Gamma, SU(2)) \) and their number by noticing that all non-abelian nilpotent subgroups of \( SU(2) \) are conjugate to a group of generalized quaternions. For instance, if \( F_n \) is the free group on \( n \) symbols and \( \Gamma_q^n \) stands for the \( q \)-th stage of the central series of \( F_n \), then \( \text{Hom}(F_n/\Gamma_q^n, SU(2)) \) has one connected component homeomorphic to \( \text{Hom}(\mathbb{Z}^n, SU(2)) \) and all the remaining homeomorphic to \( PU(2) \) (provided \( q \geq 3 \)).

The spaces \( \{\text{Hom}(F_n/\Gamma_q^n, G)\}_{n \geq 0} \) form a simplicial space with realization denoted by \( B(q,G) \), or also \( B_{com}(q,G) \) when \( q = 2 \). The space \( B(q,G) \) is the classifying space of principal \( G \)-bundles of transitional nilpotency class less than \( q \) (see [A. Adem et al., Math. Proc. Camb. Philos. Soc. 152, No. 1, 91–114 (2012; Zbl 1250.57003); Algebr. Geom. Topol. 17, No. 2, 869–893 (2017; Zbl 1360.55003)]). As a product of their work, the authors are able to obtain the mod-2 cohomology ring of \( B_{com}(Q_2^r) \), where \( Q_2^r \) is a quaternion group of order \( 2^r \). They also show that the inclusions \( B_{com}(SU(2)) \subset B(3, SU(2)) \subset \cdots \subset B(q, SU(2)) \subset \cdots \) are homology isomorphisms with coefficients over a ring where 2 is invertible.

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Baird, T., Jeffrey, L. and Selick, P., The space of commuting \((n)\)-tuples in \(\langle SU(2) \rangle\), Illinois J. Math. 55(3) (2011), 805-813. · Zbl 1278.55027


Crabb, M. C., Spaces of commuting elements in \(\langle SU(2) \rangle\), Proc. Edinburgh. Math. Soc. (2)54(1) (2011), 67-75. · Zbl 1222.55007


Okay, C., Spherical posets from commuting elements, J. Group Theory21 (2018), 593-628. · Zbl 1406.20048


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