Let $w = w(x_1, \ldots, x_k)$ be a group word, that is a nontrivial element of the free group on $x_1, \ldots, x_k$, and $G$ a group. Then, $w$ can be viewed as a $k$-variable function defined on $G$. The set of $w$-values in $G$ is denoted $G_w$ and the subgroup generated by $G_w$ is denoted $w(G)$. If $G$ is a topological group then $w(G)$ is taken to be the closed subgroup.

The concept of conciseness of a word $w$ in a class of groups $C$, that is whether the finiteness of $G_w$ implies the finiteness of $w(G)$ for all $G$ in $C$, has a long history. More recently a variation of conciseness for profinite groups has been considered. A word $w$ is strongly concise in a class $C$ of profinite groups if $|G_w| < 2^{\aleph_0}$ implies that $w(G)$ is finite for all $G$ in $C$. In this article the authors consider a closely related question. They show that, for several families of words, if you suppose $G$ is a profinite group with $|G_w| < 2^{\aleph_0}$ and $w(G)$ generated by finitely many $w$-values, then $w(G)$ is finite.

In the first theorem the authors consider words of type $[y, n v^q]$ and $[v^q, n y]$ where $v$ is the left normed commutator $[x_1, x_2, \ldots, x_k]$ and $[y, n x] = [y, x, \ldots, x]$ with $x$ repeated $n$ times and $k, n$ and $q$ all positive integers. The second theorem is more technical to state, but covers many families of words.

Reviewer: Rachel D. Camina (Cambridge)

MSC:

20E26 Residual properties and generalizations; residually finite groups
20E18 Limits, profinite groups
20F45 Engel conditions
20F10 Word problems, other decision problems, connections with logic and automata (group-theoretic aspects)

Keywords:
profinite groups; words; conciseness

Full Text: DOI

References:


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