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Discrete subsets in topological groups and countable extremally disconnected groups. (English) Zbl 0733.70.077

Summary: In 1967 Arhangel’skii posed the problem of the existence in ZFC of a nondiscrete extremally
disconnected topological group. The general case is still open, but we solve Arhangel’skii’s problem for
the class of countable groups. Namely, we prove that the existence of a countable nondiscrete extremally
disconnected group implies the existence of a rapid ultrafilter; hence, such a group cannot be constructed
in ZFC. We also prove that any countable topological group in which the filter of neighborhoods of the
identity element is not rapid contains a discrete set with precisely one limit point, which gives a negative
answer to Protasov’s question on the existence in ZFC of a countable nondiscrete group in which all
discrete subsets are closed.

MSC:
54G05  Extremally disconnected spaces, F-spaces, etc.
54H11  Topological groups (topological aspects)
03E35  Consistency and independence results
22A05  Structure of general topological groups

Full Text: DOI

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