He, Wei; Peng, Dekui; Tkachenko, Mikhail; Zhang, Heng
\(M\)-factorizability of products and \(\tau\)-fine topological groups. (English) Zbl 07354969
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Summary: Our main objective is a further study of \(M\)-factorizability in topological groups as defined in Zhang, Peng, He, Tkachenko (2020) [15]. We focus on topological-algebraic implications of \(M\)-factorizability such as \(\tau\)-precompactness, pseudo-\(\tau\)-compactness and \(\tau\)-fineness. We also study products of topological groups and present necessary and sufficient conditions on the factors guaranteeing the \(M\)-factorizability of products. Our main technical tool for this study is the new notion of \(\tau\)-fine topological group, where \(\tau > \omega\) is a cardinal. We prove the following dichotomy theorem: Every \(M\)-factorizable topological group is either \(R\)-factorizable or \(\omega_1\)-fine.

Another dichotomy is established for the product of two groups. We prove that if the product \(G \times H\) of topological groups is \(M\)-factorizable, then for every cardinal \(\tau > \omega\), either \(G\) is \(\tau\)-fine or \(H\) is pseudo-\(\tau\)-compact. We also show that the product \(G \times H\) is \(M\)-factorizable provided \(G\) is a metrizable topological group with \(w(G) \leq \tau\) and \(H\) is a \(\tau\)-fine topological group with \(hl(H) \leq \tau\).

It is also proved that the product \(G \times H\) is \(M\)-factorizable (\(R\)-factorizable) whenever \(G\) is an arbitrary \(M\)-factorizable (\(R\)-factorizable) topological group and \(H\) is a locally compact separable metrizable topological group.

MSC:
22A05 Structure of general topological groups
54A25 Cardinality properties (cardinal functions and inequalities, discrete subsets)
54H11 Topological groups (topological aspects)
54A35 Consistency and independence results in general topology

Keywords:
\(M\)-factorizability; \(R\)-factorizability; feathered group; \(\tau\)-fine group; metrizable; \(\omega\)-narrow; \(\omega\)-balanced

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References:


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