He, Wei; Peng, Dekui; Tkachenko, Mikhail; Zhang, Heng

\( \mathcal{M} \)-factorizability of products and \( \tau \)-fine topological groups. (English) [Zbl 07354969]

Topology Appl. 296, Article ID 107674, 22 p. (2021)

Summary: Our main objective is a further study of \( \mathcal{M} \)-factorizability in topological groups as defined in Zhang, Peng, He, Tkachenko (2020) [15]. We focus on topological-algebraic implications of \( \mathcal{M} \)-factorizability such as \( \tau \)-precompactness, pseudo-\( \tau \)-compactness and \( \tau \)-fineness. We also study products of topological groups and present necessary and sufficient conditions on the factors guaranteeing the \( \mathcal{M} \)-factorizability of products. Our main technical tool for this study is the new notion of \( \tau \)-fine topological group, where \( \tau > \omega \) is a cardinal. We prove the following dichotomy theorem: Every \( \mathcal{M} \)-factorizable topological group is either \( \mathbb{R} \)-factorizable or \( \omega_1 \)-fine.

Another dichotomy is established for the product of two groups. We prove that if the product \( G \times H \) of topological groups is \( \mathcal{M} \)-factorizable, then for every cardinal \( \tau > \omega \), either \( G \) is \( \tau \)-fine or \( H \) is pseudo-\( \tau \)-compact. We also show that the product \( G \times H \) is \( \mathcal{M} \)-factorizable provided \( G \) is a metrizable topological group with \( \omega(G) \leq \tau \) and \( H \) is a \( \tau \)-fine topological group with \( hl(H) \leq \tau \).

It is also proved that the product \( G \times H \) is \( \mathcal{M} \)-factorizable (\( \mathbb{R} \)-factorizable) whenever \( G \) is an arbitrary \( \mathcal{M} \)-factorizable (\( \mathbb{R} \)-factorizable) topological group and \( H \) is a locally compact separable metrizable topological group.

MSC:

22A05 Structure of general topological groups
54A25 Cardinality properties (cardinal functions and inequalities, discrete subsets)
54H11 Topological groups (topological aspects)
54A35 Consistency and independence results in general topology

Keywords:

\( \mathcal{M} \)-factorizability; \( \mathbb{R} \)-factorizability; feathered group; \( \tau \)-fine group; metrizable; \( \omega \)-narrow; \( \omega \)-balanced

Full Text: DOI

References:

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.