Toda, Yukinobu
Semiorthogonal decompositions of stable pair moduli spaces via d-critical flips. (English)

The author constructed semiorthogonal decomposition of Pandharipande-Thomas (PT for short) stable pair moduli space on Calabi-Yau (CY for short) 3-folds under certain natural assumptions, which may be viewed as a categorification of the wall-crossing formula for PT invariants in the derived category. The main tool for proving such results are d-critical birational geometry proposed in [Y. Toda, “Birational geometry for d-critical loci and wall-crossing in Calabi-Yau 3-fold”, Preprint, arXiv:1805.00182] for the study of derived category of moduli of stable objects in CY3 categories, in which context this main theorem exemplifies an analogue of D/K equivalence conjecture.

The contents in more detail:

Section 1 serves as an overview and introduction to the main content of the paper, where the author reviews standard theory of PT invariant, states the main theorem and shows its motivation.

Section 2 presents preliminary material of derived factorization categories, Fourier-Mukai functors between them as well as a version of Knörrer periodicity in this context.

In the very important section 3 the semiorthogonal decomposition $D^b(\hat{M}^+) = \langle D^b(\hat{U}), \ldots, D^b(\hat{U}), D^b(\hat{M}^-) \rangle$ is proved for a simple d-critical flip $\hat{M}^+ \rightarrow \hat{U} \leftarrow \hat{M}^-$ in completely local setting by a variant of standard semiorthogonal decomposition for simple flip in the d-critical context.

Section 4 is devoted to the globalization of the local result in the previous section. A key fact for the globalization is that the fully faithful embedding $D^b(\hat{M}^-) \hookrightarrow D^b(\hat{M}^+)$ is the Fourier-Mukai transform associated with the structure sheaf of the fiber product $\hat{M}^+ \times_{\hat{U}} \hat{M}^-$. 

Section 5 is the application of the general theory developed in section 3 and 4 to PT moduli spaces on CY 3-folds. The fact that PT moduli spaces have wall-crossing phenomena described by d-critical birational geometry was proved in the previous work [Y. Toda, “Birational geometry for d-critical loci and wall-crossing in Calabi-Yau 3-fold”, Preprint, arXiv:1805.00182]. The main theorem is also applied to noncompact examples to obtain semiorthogonal decompositions on certain (relative) Hilbert schemes and symmetric products of curves.

Section 6 gives another application of the general theory to the categorification of Kawai-Yoshioka’s formula for PT invariants on K3 surfaces.

Reviewer: Kai Xu (Cambridge)

MSC:

14N35 Gromov-Witten invariants, quantum cohomology, Gopakumar-Vafa invariants, Donaldson-Thomas invariants (algebro-geometric aspects)

14F08 Derived categories of sheaves, dg categories, and related constructions in algebraic geometry

Keywords:
derived categories of coherent sheaves; Pandharipande-Thomas invariants

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References:


[37] Reid, M.: What is a flip. Colloquium talk, Univ. of Utah


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