Basu, Saugata; Patel, Deepam

Summary: We prove a tight bound on the number of realized 0/1 patterns (or equivalently on the Vapnik-Chervonenkis codensity) of definable families in models of the theory of algebraically closed valued fields with a non-archimedean valuation. Our result improves the best known result in this direction proved by Aschenbrenner, Dolich, Haskell, Macpherson and Starchenko, who proved a weaker bound in the restricted case where the characteristics of the field $K$ and its residue field are both assumed to be 0. The bound obtained here is optimal and without any restriction on the characteristics.

We obtain the aforementioned bound as a consequence of another result on bounding the Betti numbers of semi-algebraic subsets of certain Berkovich analytic spaces, mirroring similar results known already in the case of o-minimal structure and for algebraically closed as well as algebraically closed fields. The latter result is the first result in this direction and is possibly of independent interest. Its proof relies heavily on recent results of Hrushovski and Loeser on the topology of semi-algebraic subsets of Berkovich analytic spaces.

MSC:
03C98 Applications of model theory
14F45 Topological properties in algebraic geometry

Keywords:
VC-density; Berkovich space; algebraically closed valued field; o-minimal; Betti numbers

Full Text: DOI

References:

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.