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Strongly minimal groups in o-minimal structures. (English) Zbl 07367692

B. Zilber’s trichotomy conjecture introduced in [Sibirsk. Mat. Zh. 25, 71–88 (1984; Zbl 0581.03022)] was disproved by E. Hrushovski [J. Amer. Math. Soc. 62, 147–166 (1993; Zbl 0804.03020)]. However, this conjecture is true in various restricted settings. This paper proves Zilber’s trichotomy conjecture for strongly minimal expansion of 2-dimensional groups, definable in o-minimal structures. The main theorem is as follows:

Let \( M \) be an o-minimal expansion of a real closed field, \( (G; +) \) be a 2-dimensional group definable in \( M \), and \( D = (G; +, \ldots) \) be a strongly minimal structure, all of whose atomic relations definable in \( M \). If \( D \) is not locally modular, then an algebraic closed field \( K \) is interpretable in \( D \), and the group \( G \), with all its induced \( D \)-structure, is definably isomorphic to an algebraic \( K \)-group with all its induced \( K \)-structure.

It is a generalization of [A. Hassen et al., Proc. London Math. Soc. (3) 97, 117–154 (2008; Zbl 1153.03011)] which treats the case in which \( G \) is the algebraic closure \( K = R[i] \) and \( D \) is a structure generated by an \( M \)-definable function, and its proof follows the same strategy as the Hassen’s paper; that is, constructing a field configuration and using Hrushovski’s result that a strongly minimal structure admitting a field construction interprets an algebraically closed field.

A \( D \)-definable subset of \( G^2 \) whose Morley rank is one is called a plane curve in this paper. The paper establishes the necessary ingredients for the proof of the main theorem in several distinct steps. In each step, resemblances of \( D \)-definable sets to complex algebraic sets are demonstrated including finiteness of the frontiers of plane curves, finiteness of their poles and their intersection theory. The algebraically closed field \( K \) is defined as the collection of all Jacobian matrices at zero of local smooth maps from \( G \) to \( G \) whose graph is contained in a plane curve by identifying them with the matrices in \( M_2(R) \).

Reviewer: Fujita Masato (Kure)

MSC:
14P25 Topology of real algebraic varieties
03C64 Model theory of ordered structures; o-minimality
03C45 Classification theory, stability, and related concepts in model theory

Keywords:
o-minimality; strongly minimal groups; Zilber’s conjecture

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References:


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