Let $P$ be a nonempty bounded subset in the Euclidean plane $\mathbb{R}^2$. Let us denote by $\delta(P)$ the smallest radius of a ball having its center in $P$ and covering $P$.

R. Walter proved [Minimax Theory Appl. 2, No. 2, 285-318 (2017; Zbl1380.51012)] that $L(\Gamma) \geq 2\sqrt{3} \delta(\Gamma)$ for each triangle $P$, where $\Gamma$ is the boundary of $P$ and $L(\Gamma)$ is the length of $\Gamma$. The equality holds exactly for equilateral triangles. He also conjectured $L(\Gamma) \geq \pi \delta(\Gamma)$ for any closed convex curve $\Gamma$ in $\mathbb{R}^2$, and proved the conjecture when $\Gamma$ is a convex closed curve of class $C^2$ and all curvature centers of $\Gamma$ lie in the interior of the curve.

The authors give an explicit expression of $\delta(\Gamma)$ as a function of the angles and the lengths of the sides (Theorem 2), and use it to present a simplified proof of the mentioned result for the triangle. They also prove that $L(\Gamma) \leq (2 + \pi) \delta(\Gamma)$ for any closed curve $\Gamma$ (Theorem 3), with equality exactly for sets made of the union of half-circle of radius $r$ and a line segment of length $2r$. If $P$ is a convex $n$-gon ($n \geq 2$) with boundary $\Gamma$, then (Theorem 4):

$$L(\Gamma) \leq 2 \left( 1 + (n - 1) \sin \left( \frac{\pi}{2(n-1)} \right) \right) \delta(\Gamma),$$

and an $n$-gon that achieves the equality is presented.

Finally, an open problem related to those above is proposed in the last section.

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MSC:
52A10 Convex sets in 2 dimensions (including convex curves)
52A27 Approximation by convex sets
52A40 Inequalities and extremum problems involving convexity in convex geometry
53A04 Curves in Euclidean and related spaces

Keywords:
approximation by polytopes; convex curve; convex polygon; relative Chebyshev radius; self Chebyshev radius

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References:


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