A weak homotopy equivalence type result related to Kirchberg algebras. (English)

Zbl 07377295


A purely infinite separable simple nuclear C*-algebra is called a Kirchberg algebra. In the paper under review, the authors prove a weak homotopy type result concerning two topological groups associated to a unital Kirchberg algebra.

Given such an algebra $A$, we write $A♭$ for the continuous asymptotic centralizer algebra

$$A♭ = (C^b([0,1),A))/C_0([0,1),A) \cap A'$$

and write $\text{Aut}(A \otimes \mathbb{K})$ to denote the automorphism group of the stabilization $A \otimes \mathbb{K}$ of $A$. The group of unitaries in $A♭$ is denoted by $U(A♭)$ and this is a topological group under the norm topology. Likewise, $\text{Aut}(A \otimes \mathbb{K})$ is a topological group when equipped with the point-norm topology. For each compact metrizable space $X$, the authors construct a natural homomorphism

$$\Pi_{A,X} : [X, U(A♭)] \to [X, \text{Aut}(A \otimes \mathbb{K})]$$

which is an isomorphism if $X$ is a sphere. However, the map $\Pi_{A,X}$ may not be induced by a continuous function, and thus is not a genuine weak homotopy equivalence.

The proof relies on earlier work of M. Dadarlat [J. Noncommut. Geom. 1, No. 1, 113–139 (2007; Zbl 1144.46047)] who proved that $\pi_n(\text{Aut}(A \otimes \mathbb{K}))$ is isomorphic to $KK(A, S^{n+1}A)$, where $S^kA$ denotes the $k$th suspension of $A$. The proof also uses a generalization of a homotopy theorem due to H. Nakamura [Ergodic Theory Dyn. Syst. 20, No. 6, 1749–1765 (2000; Zbl 1085.46514)], which the authors prove in the Appendix. As an application of their results, the authors compute the $K$-theory groups of $A♭$ and of the central sequence algebra $A_ω$.

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References:


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