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Numerical analysis of a self-similar turbulent flow in Bose-Einstein condensates. (English)

Summary: We study a self-similar solution of the kinetic equation describing weak wave turbulence in Bose-Einstein condensates. This solution presumably corresponds to an asymptotic behavior of a spectrum evolving from a broad class of initial data, and it features a non-equilibrium finite-time condensation of the wave spectrum \( n(\omega) \) at the zero frequency \( \omega \). The self-similar solution is of the second kind, and it satisfies boundary conditions corresponding to a nonzero constant spectrum (with all its derivative being zero) at \( \omega = 0 \) and a power-law asymptotic \( n(\omega) \to \omega^{-x} \) at \( \omega \to \infty; x \in \mathbb{R}^+ \). Finding it amounts to solving a nonlinear eigenvalue problem, i.e. finding the value \( x^* \) of the exponent \( x \) for which these two boundary conditions can be satisfied simultaneously. To solve this problem we develop a new high-precision algorithm based on Chebyshev approximations and double exponential formulas for evaluating the collision integral, as well as the iterative techniques for solving the integro-differential equation for the self-similar shape function. This procedures allow to achieve a solution with accuracy \( \approx 4.7\% \) which is realized for \( x^* \approx 1.22 \).

MSC:

35Qxx Partial differential equations of mathematical physics and other areas of application
65Dxx Numerical approximation and computational geometry (primarily algorithms)
76Fxx Turbulence

Keywords:
wave turbulence; Bose gas; nonlinear spectral problem; cubature formula; pseudospectral method; relaxation method; analysis of the error

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