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Sub-posets in $\omega^\omega$ and the strong Pytkeev$^*$ property. (English) Zbl 07387388

Topology Appl. 300, Article ID 107750, 9 p. (2021)

A partially ordered set $Q$ is a Tukey quotient of another partially ordered set $P$ iff there is a map $f : P \to Q$ which maps cofinal sets of $P$ to cofinal sets of $Q$, here a subset $R$ of $P$ is cofinal iff for every $p \in P$ there is an $r \in R$ with $p < r$. The Tukey order is then an order on partially ordered sets induced by the relation of being a quotient, that is, $P \leq_T Q$ iff $Q$ is a Tukey quotient of $P$. Tukey classes are equivalence classes of the Tukey order. The present work uses now the Tukey order to compare partially ordered subsets of the partially ordered set $(0, 1)^\omega$ (product order of $0 < 1$ for a countable product) and constructs a $2^{\aleph_0}$ sized antichain with respect to the Tukey ordering consisting of partially ordered subsets of $(0, 1)^\omega$. Furthermore, partially ordered subsets of $\omega^\omega$ are investigated.

In particular, the authors solve two recent open questions posted in the paper [J. C. Ferrando et al., Topology Appl. 208, 30–39 (2016; Zbl 1357.54017)]: First, any topological group with a $\Sigma_2$ base admits a $\omega^\omega$-base; second any separable metric space $M$ is Polish iff $\mathcal{K}(M)$ is Tukey reducible to $\Sigma$ for any unbounded and boundedly-complete proper partially ordered subset $\Sigma$ of $\omega^\omega$.

Furthermore, the authors study the strong Pytkeev$^*$ property. They provide a sufficient condition for the strong Pytkeev$^*$ property. They also investigate notions derived from a partially ordered set $P$ meeting below specifications. Recall that metrisable means that one can define a metric on the space which generates the given topology. A second countable topology is a topological space which has a countable base of the topology. Compact sets are sets satisfying that every cover of open sets of the given set contains a finite subcover. A $P$-base is defined locally: If the partially ordered set $P$ is a Tukey quotient of some base of the neighbourhood of $x$, then one says that the point $x$ has a $P$-base and the whole space $X$ has a $P$-base, if every $x \in X$ has a $P$-base. The authors extend Theorem 1.2 of [T. Banakh, J. Kakol and J. P. Schürz, $\omega^\omega$-base and infinite-dimensional compact sets in locally convex spaces, to appear in Rev. Mat. Complut. (doi:10.1007/s13163-021-00397-9), Preprint: arXiv:2007.04420] by showing that each uncountably-dimensional locally convex space with a $P$-base contains an infinite-dimensional metrizable compact subspace if $P$ is a directed set equipped with a second-countable topology in which every convergent sequence in $P$ is bounded.

Reviewer: Frank Stephan (Singapore)

MSC:
54D70 Base properties of topological spaces
06A06 Partial orders, general
46B50 Compactness in Banach (or normed) spaces

Keywords:
Tukey order; strong Pytkeev$^*$ property; $\omega^\omega$-base; $\mathcal{K}(M)$-base; $P$-base; locally convex space (lcs); partially ordered sets (posets); function spaces

Full Text: DOI

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