Summary: A topological group $G$ with $|G| > 1$ is called $d$-independent if for every subgroup $S$ of $G$ with $|S| < 2^\omega$, one can find a countable dense subgroup $H$ of $G$ such that $S \cap H = \{e\}$. Therefore, $d$-independent groups are separable and have cardinality at least $2^\omega$. Our main result is a purely algebraic characterization of $d$-independence in the class of compact metrizable abelian groups. We prove that a compact metrizable abelian group $G$ with $|G| > 1$ is $d$-independent if and only if for every integer $m \geq 1$, either $|mG| = 2^\omega$ or $|mG| = 1$. This characterization implies that a compact metrizable abelian group is $d$-independent if and only if it is maximally fragmentable [Comfort and Dikranjan (2014) [4]] iff $G$ an $M$-group as defined by Dikranjan and Shakhmatov (2016) in [7].

Also we present a characterization of separable metrizable $d$-independent abelian groups and show that products of separable topological groups can often be $d$-independent, even if the factors fail to be $d$-independent.

MSC:

22A05 Structure of general topological groups
22C05 Compact groups
54H11 Topological groups (topological aspects)

Keywords:

compact abelian group; $M$-group; $d$-independent; separable metrizable; independent subset; locally compact group

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References:


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