Summary: We propose a novel collocation method with Radial Basis Functions for the solution of the inhomogeneous parabolic equation \((∂t + L)u(·, t) = f\) on \(Ω \subseteq \mathbb{R}^d\), with \(L\) an elliptic operator. As original contribution, we rewrite the solution in terms of the exponential operator \(\exp(-tL)\), which is then computed through the Padé-Chebyshev approximation of the 1D Gaussian function. The resulting meshless solver uniformly converges to the ground-truth solution, as the degree of the rational polynomial increases, and is independent of the evaluation of the spectrum of \(L\) (i.e., spectrum-free), of the discretisation of the temporal derivative, and of user-defined parameters. Since the solution is approximated as a linear combination of Radial Basis Functions, we study the conditions on the generating kernel that guarantee the \(L\)-differentiability of the meshless solution. In our tests, we compare the proposed meshless and spectrum-free solvers with the meshless spectral eigen-decomposition and the meshless \(θ\)-method on the heat equation in a transient regime. With respect to these previous works, at small scales the Padé-Chebyshev method has a higher numerical stability and approximation accuracy, which are expressed in terms of the selected degree of the rational polynomial and of the spectral properties of the matrix that discretises the parabolic operator.

MSC:

65Nxx Numerical methods for partial differential equations, boundary value problems
65Mxx Numerical methods for partial differential equations, initial value and time-dependent initial-boundary value problems
41Axx Approximations and expansions

Keywords:
parabolic PDEs; heat equation; collocation methods; radial basis functions; spectrum; spectrum-free solvers

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References:
  · Zbl 0917.65063 · doi:10.1145/285861.285868

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