Summary: In 2005, Boman et al. introduced the concept of factor width for a real symmetric positive semidefinite matrix. This is the smallest positive integer $k$ for which the matrix $A$ can be written as $A = VV^T$ with each column of $V$ containing at most $k$ non-zeros. The cones of matrices of bounded factor width give a hierarchy of inner approximations to the PSD cone. In the polynomial optimization context, a Gram matrix of a polynomial having factor width $k$ corresponds to the polynomial being a sum of squares of polynomials of support at most $k$. Recently, Ahmadi and Majumdar [1], explored this connection for case $k = 2$ and proposed to relax the reliance on polynomials that are sums of squares in semidefinite programming to polynomials that are sums of binomial squares. In this paper, we prove some results on the geometry of the cones of matrices with bounded factor widths and their duals, and use them to derive new results on the limitations of certificates of nonnegativity of quadratic forms by sums of $k$-nomial squares using standard multipliers. In particular we show that they never help for symmetric quadratics, for any quadratic if $k = 2$, and any quaternary quadratic if $k = 3$. Furthermore we give some evidence that those are a complete list of such cases.

MSC:
13J30 Real algebra
12D15 Fields related with sums of squares (formally real fields, Pythagorean fields, etc.)
90C30 Nonlinear programming

Keywords:
factor width; sums of squares; positive semidefinite; $K$-nomials; scaled diagonally dominant sum of squares (SDSOS)

Full Text: DOI

References:


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