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Hölder parameterization of iterated function systems and a self-affine phenomenon. (English)
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Summary: We investigate the Hölder geometry of curves generated by iterated function systems (IFS) in a complete metric space. A theorem of Hata from 1985 asserts that every connected attractor of an IFS is locally connected and path-connected. We give a quantitative strengthening of Hata’s theorem. First we prove that every connected attractor of an IFS is \((1/s)\)-Hölder path-connected, where \(s\) is the similarity dimension of the IFS. Then we show that every connected attractor of an IFS is parameterized by a \((1/\alpha)\)-Hölder curve for all \(\alpha > s\). At the endpoint, \(\alpha = s\), a theorem of Remes from 1998 already established that connected self-similar sets in Euclidean space that satisfy the open set condition are parameterized by \((1/s)\)-Hölder curves. In a secondary result, we show how to promote Remes’ theorem to self-similar sets in complete metric spaces, but in this setting require the attractor to have positive \(s\)-dimensional Hausdorff measure in lieu of the open set condition. To close the paper, we determine sharp Hölder exponents of parameterizations in the class of connected self-affine Bedford-McMullen carpets and build parameterizations of self-affine sponges. An interesting phenomenon emerges in the self-affine setting. While the optimal parameter \(s\) for a self-similar curve in \(\mathbb{R}^n\) is always at most the ambient dimension \(n\), the optimal parameter \(s\) for a self-affine curve in \(\mathbb{R}^n\) may be strictly greater than \(n\).

MSC:
28A80 Fractals
26A16 Lipschitz (Hölder) classes
28A75 Length, area, volume, other geometric measure theory
53A04 Curves in Euclidean and related spaces

Keywords:
Hölder curves; parameterization; iterated function systems; self-affine sets

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References: