Ahmed, H. M.
Computing expansions coefficients for Laguerre polynomials. (English) Zbl 07394784

In the paper the author develops identities that allow us expressing $x^m p_n(x)$, $p_n(x)q_m(x)$, and $\prod_{j=1}^{s} p_{n_j}(x)$ in terms of generalized Laguerre polynomials, denoted by $L_n^\gamma$, where $p_n(x)$, $q_m(x)$ and $p_{n_j}(x)$ are polynomials of degrees $n$, $m$ and $n_j$, respectively.

More precisely, the author shows that in the expansion

$$x^m q_m(x) = \sum_{i=0}^{n+m} a_i(n,m)L_i^\alpha(x),$$

the coefficients $a_i(n,m)$, can be written as,

$$a_i(n,m) = (-1)^i \frac{i!((\alpha + 1)_m)}{i!} \sum_{r=0}^{n} \binom{r + m}{i} \frac{(\alpha + m + 1)_i}{r!} q_n(0),$$

for each $i = 0, \ldots, m + n$. So, as corollary, the cases $x^m L_n^\gamma(x)$, $x^m P_n^\gamma(x)$, $x^m V_n(x)$, $x^m W_n(x)$ (where $L_n^\gamma$, $P_n^\gamma$, $V_n$ and $W_n$ denote the generalized Laguerre polynomials, the Jacobi polynomials, the Chebyshev polynomials of the third and fourth kinds, respectively), are studied and explicit expansions are obtained for $a_i(n,m)$.

Later, in the identity,

$$p_n(x)q_m(x) = \sum_{i=0}^{n+m} c_i(n,m)L_i^\alpha(x),$$

the author states that

$$c_i(n,m) = \frac{(-1)^i i!}{(\alpha + 1)_i} \sum_{k=0}^{m} \sum_{r=0}^{n} \frac{k + r}{i} (\alpha + 1)_{r+k} q^k_m(0)p^r_n(0),$$

$i = 0, \ldots, n + m$. Again, cases such as $L_n^\gamma(x)L_m^\beta(x)$ and $C_n^\beta(x)C_m^\lambda(x)$, (the Charlier polynomials), are considered among others.

Finally, in the expansion

$$\prod_{j=1}^{s} p_{n_j}(x) = \sum_{i=1}^{N_s} c_i(n_1, \ldots, n_s)L_i^\alpha(x),$$

where $N_s = \sum_{j=1}^{s} n_j$, the coefficients $c_i(n_1, \ldots, n_s)$ satisfy the identity

$$c_i(n_1, \ldots, n_s) = \frac{(-1)^i i!}{(\alpha + 1)_i} \sum_{r_1=0}^{n_1} \cdots \sum_{r_s=0}^{n_s} \frac{d_s}{i} (\alpha + 1)_d \prod_{j=1}^{s} \frac{p_{n_j}(0)}{r_j!},$$

$i = 0, \ldots, N_s$, and $d_s = r_1 + \cdots + r_s(x)$.

Once again, the author considers $p_{n_j}(x) = H_{n_j}(x)$ (Hermite polynomials), $p_{n_j}(x) = C_{n_j}^{(\lambda)}(x)$ and $p_{n_j}(x) = L_{n_j}^\gamma(x)$ and obtains, as corollary, explicit expansions for coefficients $c_i(n_1, \ldots, n_s)$.

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MSC:

- 42C10 Fourier series in special orthogonal functions (Legendre polynomials, Walsh functions, etc.)
- 33B45 Basic orthogonal polynomials and functions (Askey-Wilson polynomials, etc.)
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orthogonal polynomials; expansions of polynomials; linearization and connection problems; generalized hypergeometric functions

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