Summary: The aim of this paper is to provide a self-contained proof of a general case of the coarea inequality, also known as the Eilenberg inequality. The result is known, but we are not aware of any place that a proof would be written with all details. The known proof is based on a difficult result of Davies. Our proof is elementary and does not use Davies’ theorem. Instead we use an elegant argument that we learned from Nazarov through MathOverflow. We also obtain some generalizations of the coarea inequality.

MSC:

28A25 Integration with respect to measures and other set functions
28A75 Length, area, volume, other geometric measure theory
28A78 Hausdorff and packing measures

Keywords:

metric spaces; Hausdorff measure; coarea inequality

Full Text: DOI

References:
