Summary: We propose a reconstruction-based \textit{a posteriori} error estimate for linear advection problems in one space dimension. In our framework, a stable variational ultra-weak formulation is adopted, and the equivalence of the $L^2$-norm of the error with the dual graph norm of the residual is established. This dual norm is showed to be localizable over vertex-based patch subdomains of the computational domain under the condition of the orthogonality of the residual to the piecewise affine hat functions. We show that this condition is valid for some well-known numerical methods including continuous/discontinuous Petrov-Galerkin and discontinuous Galerkin methods. Consequently, a well-posed local problem on each patch is identified, which leads to a global conforming reconstruction of the discrete solution. We prove that this reconstruction provides a guaranteed upper bound on the $L^2$ error. Moreover, up to a generic constant, it also gives local lower bounds on the $L^2$ error, where the constant only depends on the mesh shape-regularity. This, in particular, leads to robustness of our estimates with respect to the polynomial degree. All the above properties are verified in a series of numerical experiments, additionally leading to asymptotic exactness. Motivated by these results, we finally propose a heuristic extension of our methodology to any space dimension, achieved by solving local least-squares problems on vertex-based patches. Though not anymore guaranteed, the resulting error indicator is still numerically robust with respect to both advection velocity and polynomial degree in our collection of two-dimensional test cases including discontinuous solutions aligned and not aligned with the computational mesh.

MSC:

- 65N15 Error bounds for boundary value problems involving PDEs
- 65N30 Finite element, Rayleigh-Ritz and Galerkin methods for boundary value problems involving PDEs
- 35F05 Linear first-order PDEs

Keywords:

- linear advection problem;
- discontinuous Galerkin method;
- Petrov-Galerkin method;
- \textit{a posteriori} error estimate;
- local efficiency;
- advection robustness;
- polynomial-degree robustness

Full Text: DOI

References:


