Categories of orthogonality spaces. (English) Zbl 07396436

Summary: An orthogonality space is a set equipped with a symmetric and irreflexive binary relation. We consider orthogonality spaces with the additional property that any collection of mutually orthogonal elements gives rise to the structure of a Boolean algebra. Together with the maps that preserve the Boolean substructures, we are led to the category $NOS$ of normal orthogonality spaces.

Moreover, an orthogonality space of finite rank is called linear if for any two distinct elements $e$ and $f$ there is a third one $g$ such that exactly one of $f$ and $g$ is orthogonal to $e$ and the pairs $e, f$ and $e, g$ have the same orthogonal complement. Linear orthogonality spaces arise from finite-dimensional Hermitian spaces. We are led to the full subcategory $LOS$ of $NOS$ and we show that the morphisms are the orthogonality-preserving lineations.

Finally, we consider the full subcategory $EOS$ of $LOS$ whose members arise from positive definite Hermitian spaces over Baer ordered $*$-fields with a Euclidean fixed field. We establish that the morphisms of $EOS$ are induced by generalised semiunitary mappings.

MSC:

81P10 Logical foundations of quantum mechanics; quantum logic (quantum-theoretic aspects)
06C15 Complemented lattices, orthocomplemented lattices and posets
46C05 Hilbert and pre-Hilbert spaces: geometry and topology (including spaces with semidefinite inner product)

Keywords:
orthogonality spaces; undirected graphs; categories; Boolean subalgebras; linear orthogonality spaces; generalised semilinear map

Full Text: DOI

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