Tkachenko, Mikhail
On C-embedded subspaces of weakly Lindelöf spaces. (English) Zbl 07402616
Topology Appl. 302, Article ID 107822, 12 p. (2021)

In this well-written paper the author studies dense C-embedded subspaces of completely regular, weakly Lindelöf spaces. In particular, he is trying to find out under which conditions such subspaces are also weakly Lindelöf. Some of the interesting results are mentioned below.

In Section 2 the author shows that a dense subspace \( Y \) of a product space \( X = \prod_{i \in I} X_i \) is weakly Lindelöf if and only if \( \pi_I(Y) \) is weakly Lindelöf for every finite \( J \subset I \), where \( \pi_I \) is the projection of \( X \) onto \( \prod_{i \in J} X_i \). Therefore, if the product space \( X \) is weakly Lindelöf and \( Y \subseteq X \) is such that \( \pi_J(Y) = \prod_{i \in J} X_i \), for every finite \( J \subset I \), then \( Y \) is weakly Lindelöf. Then, it follows from results in Chapter 10 of the book [W. W. Comfort and S. Negrepontis, Chain conditions in topology. Cambridge: Cambridge University Press (1982; Zbl 0488.54002)] that if, in addition, \( \pi_J(Y) = \prod_{i \in J} X_i \), for every countable \( J \subset I \), then \( Y \) is also \( C \)-embedded in \( X \). As a corollary of these results the author shows that if \( X \) is the product of a family of completely regular spaces of countable pseudocharacter and \( X \) is weakly Lindelöf, then every dense \( C \)-embedded subspace \( Y \) of \( X \) is also weakly Lindelöf.

In Section 3 the author proves that every dense \( C^\ast \)-embedded subspace of an arbitrary product \( \prod = \prod_{i \in J} X_i \) of Eberlein compacta is pseudocompact (hence \( C \)-embedded in \( \Pi \)) and weakly Lindelöf. Examples are given to show that some possible conjectures are not true and several interesting open problems are stated.

Reviewer: Ivan S. Gotchev (New Britain)

MSC:
54B10 Product spaces in general topology
54C45 \( C \)- and \( C^\ast \)-embedding
54D20 Noncompact covering properties (paracompact, Lindelöf, etc.)

Keywords:
compact; pseudocompact; weakly Lindelöf; pseudo-\( \omega_1 \)-compact; \( C \)-embedded; \( C^\ast \)-embedded; Eberlein compacta; topological group

Full Text: DOI

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