Summary: We present and prove closed form expressions for some families of binomial determinants with signed Kronecker deltas that are located along an arbitrary diagonal in the corresponding matrix. They count cyclically symmetric rhombus tilings of hexagonal regions with triangular holes. We extend a previous systematic study of these families, where the locations of the Kronecker deltas depended on an additional parameter, to families with negative Kronecker deltas. By adapting Zeilberger’s holonomic ansatz to make it work for our problems, we can take full advantage of computer algebra tools for symbolic summation. This, together with the combinatorial interpretation, allows us to realize some new determinantal relationships. From there, we are able to resolve all remaining open conjectures related to these determinants, including one from 2005 due to Lascoux and Krattenthaler.

MSC:

- 15A15 Determinants, permanents, traces, other special matrix functions
- 05A10 Factorials, binomial coefficients
- 05C50 Graphs and linear algebra (matrices, eigenvalues, etc.)
- 11C20 Matrices, determinants in number theory
- 05B45 Combinatorial aspects of tessellation and tiling problems
- 52C20 Tilings in 2 dimensions (aspects of discrete geometry)

Keywords:

- binomial determinants
- Zeilberger’s holonomic ansatz
- rhombus tilings

References:

[14] Koutschan, Christoph, Advanced applications of the holonomic systems approach (2009), Johannes Kepler University: Johannes Kepler University Linz, Austria, (Ph.D. thesis) · Zbl 1344.68301
Linz, Austria, http://www.risc.jku.at/research/combinat/software/HolonomicFunctions/


[27] Wong, Elaine, Electronic material accompanying this article (2021), Available at https://wongey.github.io/binom-det/


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