Summary: Let $\pi : \mathbb{R}^n \rightarrow \mathbb{R}^d$ be any linear projection, let $A$ be the image of the standard basis. Motivated by Postnikov’s study of positive Grassmannians via plabic graphs and Galashin’s connection of plabic graphs to slices of zonotopal tilings of 3-dimensional cyclic zonotopes, we study the poset of subdivisions induced by the restriction of $\pi$ to the $k$-th hypersimplex, for $k = 1, \ldots, n-1$. We show that: For arbitrary $A$ and for $k \leq d + 1$, the corresponding fiber polytope $F(k)(A)$ is normally isomorphic to the Minkowski sum of the secondary polytopes of all subsets of $A$ of size $\max\{d+2, n-k+1\}$. When $A = P_n$ is the vertex set of an $n$-gon, we answer the Baues question in the positive: the inclusion of the poset of $\pi$-coherent subdivisions into the poset of all $\pi$-induced subdivisions is a homotopy equivalence. When $A = C(d, n)$ is the vertex set of a cyclic $d$-polytope with $d$ odd and any $n \geq d + 3$, there are non-lifting (and even more so, non-separated) $\pi$-induced subdivisions for $k = 2$.

MSC:

52B20 Lattice polytopes in convex geometry (including relations with commutative algebra and algebraic geometry)

52B45 Dissections and valuations (Hilbert’s third problem, etc.)

52C22 Tilings in $n$ dimensions (aspects of discrete geometry)

52C40 Oriented matroids in discrete geometry

51M20 Polyhedra and polytopes; regular figures, division of spaces

05C10 Planar graphs; geometric and topological aspects of graph theory

05C15 Coloring of graphs and hypergraphs

Keywords:

hypersimplex; subdivisions; fiber polytope; Baues problem; separated sets; Zitat Galashin: 1406.52039; plabic graphs; plabic=planar bicolored

Full Text: DOI

References:


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