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Nonnegative and strictly positive linearization of Jacobi and generalized Chebyshev polynomials. (English) [Zbl 07424763]
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Summary: In the theory of orthogonal polynomials, as well as in its intersection with harmonic analysis, it is an important problem to decide whether a given orthogonal polynomial sequence \((P_n(x))_{n \in \mathbb{N}_0}\) satisfies nonnegative linearization of products, i.e., the product of any two \(P_m(x), P_n(x)\) is a conical combination of the polynomials \(P_{m-n}(x), \ldots, P_{m+n}(x)\). Since the coefficients in the arising expansions are often of cumbersome structure or not explicitly available, such considerations are generally very nontrivial. Gasper (Can J Math 22:582-593, 1970) was able to determine the set \(V\) of all pairs \((\alpha, \beta) \in (-1, \infty)^2\) for which the corresponding Jacobi polynomials \((R_n^{(\alpha,\beta)}(x))_{n \in \mathbb{N}_0}\), normalized by \(R_0^{(\alpha,\beta)}(1) \equiv 1\), satisfy nonnegative linearization of products. Szwarc (Inzell Lectures on Orthogonal Polynomials, Adv. Theory Spec. Funct. Orthogonal Polynomials, vol 2, Nova Sci. Publ., Hauppauge, NY pp 103-139, 2005) asked to solve the analogous problem for the generalized Chebyshev polynomials \((T_n^{(\alpha,\beta)}(x))_{n \in \mathbb{N}_0}\), which are the quadratic transformations of the Jacobi polynomials and orthogonal w.r.t. the measure \((1-x^2)^\alpha |x|^{2\beta+1} \chi_{(-1,1)}(x)dx\).

In this paper, we give the solution and show that \((T_n^{(\alpha,\beta)}(x))_{n \in \mathbb{N}_0}\) satisfies nonnegative linearization of products if and only if \((\alpha, \beta) \in V\), so the generalized Chebyshev polynomials share this property with the Jacobi polynomials. Moreover, we reconsider the Jacobi polynomials themselves, simplify Gasper’s original proof and characterize strict positivity of the linearization coefficients. Our results can also be regarded as sharpenings of Gasper’s one.

MSC:
33C45 Orthogonal polynomials and functions of hypergeometric type (Jacobi, Laguerre, Hermite, Askey scheme, etc.)
33C47 Other special orthogonal polynomials and functions
42C10 Fourier series in special orthogonal functions (Legendre polynomials, Walsh functions, etc.)

Keywords:
Jacobi polynomials; generalized Chebyshev polynomials; Fourier expansions; nonnegative linearization; strictly positive linearization; linearization coefficients

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References:

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