A semitopological semigroup is a semigroup with a Hausdorff topology for which the product is separately continuous. We say that semitopological semigroup $S$ is left reversible if $s\bar{S} \cap t\bar{S} \neq \emptyset$ for all $s, t \in S$. A subset $K$ of a Banach space is said to have normal structure if for each bounded subset $W$ of $K$ that contains more than one point, there is $w$ in convex hull of $W$ such that $\sup\{\|x - w\| : w \in W\} < \sup\{\|x - y\| : x, y \in W\}$. We say that $K$ has weak normal structure (weak* normal structure) if every weakly compact (weak* compact) subset $C$ of $K$ has normal structure. A Banach space $E$ is called $L$-embedded if the canonical image of $E$ in its second dual $E^{**}$ is $l^1$-summand in $E^{**}$, i.e. if there is a subspace $\sum$ of $E^{**}$ such that $E^{**} = E \oplus_1 \sum$.

In this paper, the authors study the long standing problem as to when a left reversible semitopological semigroup $S$ acting as a nonexpansive mapping on a weak* closed convex subset $K$ of the dual space $E^*$ of a Banach space $E$ has a common fixed point in $K$. It is shown that if the action is separately weak* continuous and there is an element $b \in K$ such that orbit $Sb$ is bounded, then $K$ has a common fixed point for $S$ provided $K$ has weak* normal structure. If $K$ is also $L$-embedded and $Sb$ is weakly precompact, then $K$ has a common fixed point for $S$ provided $K$ has a weak normal structure. The authors also study the notion of local amenability on the space $C_b(S)$ of all bounded continuous functions on a semitopological semigroup $S$.

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47D03 Groups and semigroups of linear operators
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