Wang, Rongxiang; Yan, Lijun
Global boundedness in a nonlinear attraction-repulsion chemotaxis system with nonlinear productions. (Chinese. English summary) Zbl 07448785

Summary: In this paper, we study the attraction-repulsion chemotaxis system with nonlinear productions and logistic source,
\[ \begin{align*}
    u_t &= \nabla \cdot (D(u)\nabla u) - \nabla \cdot (\Phi(u)\nabla v) + \nabla \cdot (\Psi(u)\nabla w) + f(u), \\
    0 &= \Delta v + \alpha u^k - \beta v, \\
    0 &= \Delta w + \gamma u^l - \delta w,
\end{align*} \]
in bounded domain \( \Omega \subset \mathbb{R}^n \) \((n \geq 1)\), subject to the homogeneous Neumann boundary conditions, \( D, \Phi, \Psi \in C^2[0, +\infty) \) nonnegative, with \( D(s) \geq (S+1)^p \) for \( s \geq 0 \), \( \Phi(s) \leq \chi s^q \), \( \Psi(s) \geq \xi s^g \) for \( s > 1 \), and \( f \) satisfying \( f(s) \leq s(a - bs^d) \) for \( s > 0 \), \( f(0) \geq 0 \). It is proved that if the attraction is dominated by one of the repulsion, logistic source and the nonlinear diffusion mechanisms with \( q+k < \max\{g+l, d+1, \frac{2}{n} + p + 1\} \), then the solutions would be globally bounded. Moreover, under the three balance situations with \( q+k = g+l = d+1 \), \( q+k = g+l > d+1 \) or \( q+k = d+1 > g+l \), the boundedness of solutions would be determined by the sizes of the coefficients involved.

MSC:
35Q92 PDEs in connection with biology, chemistry and other natural sciences
35M30 Mixed-type systems of PDEs

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nonlinearity; chemotaxis; logistic source; attraction-repulsion; global boundedness