Summary: The goal of this paper is the study of the integrability of the geodesic flow on \( k \)-step nilpotent Lie groups, \( k = 2, 3 \), when equipped with a left-invariant metric. Liouville integrability is proved in low dimensions. Moreover, it is shown that complete families of first integrals can be constructed with Killing vector fields and symmetric Killing 2-tensor fields. This holds for dimension \( m \leq 5 \). The situation in dimension six is similar in most cases. Several algebraic relations on the Lie algebra of first integrals are explicitly written. Also invariant first integrals are analyzed and several involution conditions are shown.

MSC:

53D25 Geodesic flows in symplectic geometry and contact geometry
70H06 Completely integrable systems and methods of integration for problems in Hamiltonian and Lagrangian mechanics
22E25 Nilpotent and solvable Lie groups
70G65 Symmetries, Lie group and Lie algebra methods for problems in mechanics
70H05 Hamilton’s equations
22E60 Lie algebras of Lie groups

Keywords:

geodesic flow; first integrals; Liouville integrability; nilpotent Lie groups; Killing tensor fields

References:


Edited by FIZ Karlsruhe, the European Mathematical Society and the Heidelberg Academy of Sciences and Humanities © 2022 FIZ Karlsruhe GmbH