Summary: In geometric, algebraic, and topological combinatorics, the unimodality of combinatorial generating polynomials is frequently studied. Unimodality follows when the polynomial is (real) stable, a property often deduced via the theory of interlacing polynomials. Many of the open questions on stability and unimodality of polynomials pertain to the enumeration of faces of cell complexes. In this paper, we relate the theory of interlacing polynomials to the shellability of cell complexes. We first derive a sufficient condition for stability of the $h$-polynomial of a subdivision of a shellable complex. To apply it, we generalize the notion of reciprocal domains for convex embeddings of polytopes to abstract polytopes and use this generalization to define the family of stable shellings of a polytopal complex. We characterize the stable shellings of cubical and simplicial complexes, and apply this theory to answer a question of F. Brenti and V. Welker [Math. Z. 259, No. 4, 849–865 (2008; Zbl 1158.52013)] on barycentric subdivisions for the well-known cubical polytopes. We also give a positive solution to a problem of F. Mohammadi and V. Welker [Lect. Notes Math. 2176, 77–122 (2017; Zbl 1369.05225)] on edgewise subdivisions of cell complexes. We end by relating the family of stable line shellings to the combinatorics of hyperplane arrangements. We pose related questions, answers to which would resolve some long-standing problems while strengthening ties between the theory of interlacing polynomials and the combinatorics of hyperplane arrangements.

MSC:

05E45 Combinatorial aspects of simplicial complexes
52B20 Lattice polytopes in convex geometry (including relations with commutative algebra and algebraic geometry)

Keywords:

shellability; polytopal complex; polytope; subdivision; real-rooted; unimodal

Full Text: DOI

References:
