Ishii, Yuta
Stability analysis of spike solutions to the Schnakenberg model with heterogeneity on metric graphs. (English) [Zbl 07451432]
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Summary: In this paper, we consider the linear stability of spiky stationary solutions on compact metric graphs for the Schnakenberg model with heterogeneity. The existence of spiky solutions has been shown by the author and Kurata in the work (Ishii and Kurata 2021). By studying the associated linearized eigenvalue problem, we establish the abstract theorem on the stability of the solutions for general compact metric graphs. In particular, the associated Green’s function plays an important role in calculating eigenvalues, and we reveal the several needed conditions for Green’s function on general graphs. To show the stability, we calculate two eigenvalues of order $O(1)$ and of order $o(1)$, respectively. The stability of eigenvalues of order $O(1)$ is shown by using the lemma of Wei and Winter for non-local eigenvalue problem. The stability of eigenvalues of order $o(1)$ is determined by the interaction of the heterogeneity with Green’s function. Moreover, based on the abstract theorem, we give precise stability thresholds with respect to diffusion constants for the solutions without heterogeneity function on the $Y$-shaped graph and the $H$-shaped graph. In particular, compared with the one-dimensional interval case, we obtain new phenomena on the stability of two-peak solutions by the effect of the geometry of these concrete graphs. In addition, we also present the effect of heterogeneity by using a typical example.

MSC:
35B35 Stability in context of PDEs
35R02 PDEs on graphs and networks (ramified or polygonal spaces)
35K57 Reaction-diffusion equations
35Q92 PDEs in connection with biology, chemistry and other natural sciences

Keywords:
pattern formation; Schnakenberg model; metric graph; spike solution; stability

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References:

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