The author has as his main theme the dependence of solutions to the Beltrami equation (*) $F_{\bar{z}} = \mu F_z$ and connections to Teichmüller theory and harmonic analysis. He considers the function $\mu(z)$ with $\|\mu\|_{\infty} < 1$, sometimes with compact support, and lets $F^\mu$ be a “normalized” solution of (*). Theorem 1 shows that the map $\mu \rightarrow \log(F^\mu_z)$ is a complex holomorphic map of BMO($\mathbb{R}^2$). This is an extension of a well-known result of H. M. Reimann [Comment. Math. Helv. 49, 260-276 (1974; Zbl 0289.30027)] but uses careful analysis of the argument of $(F^\mu)_z$, via approximation.

An open set $\Omega \ni \infty$ has a univalence criterion if there is an $a = a(\Omega)$ such that if $g$ is analytic in $\Omega$ with $|g(z)| = o(1)$ at $\infty$ and $|g'(z)|\text{dist}(z, \partial \Omega) < \infty$ then $g$ is one-to-one. Theorem 2 asserts that this is equivalent to several things, one being that $g$ has a representation as a Hilbert transform of a function $h \in L^\infty(\Omega^c)$, and makes contact with the improved Thurston- Sullivan $\lambda$-lemma [cf. L. Bers and H. L. Royden, Acta Math. 157, 259-286 (1986; Zbl 0619.30027)]. These results require no regularity of $\partial \Omega$.

Analogues of some results are given for VMO. The paper is compactly written, and has a few inessential typographical errors.

MSC:
- 30C65 Quasiconformal mappings in $\mathbb{R}^n$, other generalizations
- 30D55 $H^p$-classes (MSC2000)
- 30F60 Teichmüller theory for Riemann surfaces

Keywords:
- Beltrami equation; Teichmüller theory; VMO

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