Summary: We consider a planar convex body $C$ and we prove several analogs of Roth’s theorem on irregularities of distribution. When $\partial C$ is $C^2$ regardless of curvature, we prove that for every set $\mathcal{P}_N$ of $N$ points in $\mathbb{T}^2$ we have the sharp bound
\[
\int_0^1 \int_{\mathbb{T}^2} \left| \text{card}(\mathcal{P}_N \cap (\tau C + t)) - \tau^2 N |C|| \right|^2 dt d\tau \geq c N^{1/2}.
\]
When $\partial C$ is only piecewise $C^2$ and is not a polygon we prove the sharp bound
\[
\int_0^1 \int_{\mathbb{T}^2} \left| \text{card}(\mathcal{P}_N \cap (\tau C + t)) - \tau^2 N |C|| \right|^2 dt d\tau \geq c N^{2/5}.
\]
We also give a whole range of intermediate sharp results between $N^{2/5}$ and $N^{1/2}$. Our proofs depend on a lemma of Cassels-Montgomery, on ad hoc constructions of finite point sets, and on a geometric type estimate for the average decay of the Fourier transform of the characteristic function of $C$.

MSC:

11K38 Irregularities of distribution, discrepancy
42B10 Fourier and Fourier-Stieltjes transforms and other transforms of Fourier type

Keywords:
irregularities of distribution; geometric discrepancy; Roth’s theorem; Fourier transforms; Cassels-Montgomery lemma; inner disk condition

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References:
