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**Exact solutions to a nonlinear partial differential equation: the Product-of-Curvatures Poisson \((u_{xx}u_{yy} = 1)\).** (English) Zbl 07472414


Summary: We analytically and numerically solve the PCP equation, \(u_{xx}u_{yy} = 1\), with homogeneous Dirichlet boundary conditions on the unit square. Chebyshev and Fourier spectral methods with low degree truncations yield moderate accuracy but the usual exponential rate of convergence of spectral methods is destroyed by the boundary singularities of the solution. In the sequel to this work, we will apply a variety of strategies including a change-of-coordinates and singular basis functions to recover spectral accuracy in spite of the boundary singularities. As preparation for this numerical study, we find explicit solutions to related problems to the two-dimensional PCP equation in a domain with a boundary that is an ellipse and the three-dimensional PCP equation in a cubic domain. We also analyze the boundary behavior of these solutions: all have complicated singularities with unbounded first derivatives.

**MSC:**
- 33F05 Numerical approximation and evaluation of special functions
- 42A10 Trigonometric approximation
- 41A50 Best approximation, Chebyshev systems

**Keywords:**
- explicit solutions
- pseudospectral
- Chebyshev polynomials
- boundary singularities
- corner singularities

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**References:**


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