Summary: We have the best constants of three kinds of discrete Sobolev inequalities on the complete bipartite graph with $2N$ vertices, that is, $K_{N,N}$. We introduce a discrete Laplacian $A$ on $K_{N,N}$. $A$ is a $2N \times 2N$ real symmetric positive-semidefinite matrix whose eigenvector corresponding to zero eigenvalue is $1 = (1, 1, \ldots, 1) \in \mathbb{C}^{2N}$. A discrete heat kernel, a Green’s matrix and a pseudo Green’s matrix play important roles in giving the best constants.

MSC:

46E39 Sobolev (and similar kinds of) spaces of functions of discrete variables
35K08 Heat kernel

Keywords:
discrete Sobolev inequality; discrete Laplacian; Green’s matrix; reproducing relation

Full Text: DOI