Zhang, Chao; Kokoszka, Piotr; Petersen, Alexander
Wasserstein autoregressive models for density time series. (English) Zbl 07476226
J. Time Ser. Anal. 43, No. 1, 30-52 (2022)

Summary: Data consisting of time-indexed distributions of cross-sectional or intraday returns have been extensively studied in finance, and provide one example in which the data atoms consist of serially dependent probability distributions. Motivated by such data, we propose an autoregressive model for density time series by exploiting the tangent space structure on the space of distributions that is induced by the Wasserstein metric. The densities themselves are not assumed to have any specific parametric form, leading to flexible forecasting of future unobserved densities. The main estimation targets in the order-$p$ Wasserstein autoregressive model are Wasserstein autocorrelations and the vector-valued autoregressive parameter. We propose suitable estimators and establish their asymptotic normality, which is verified in a simulation study. The new order-$p$ Wasserstein autoregressive model leads to a prediction algorithm, which includes a data driven order selection procedure. Its performance is compared to existing prediction procedures via application to four financial return data sets, where a variety of metrics are used to quantify forecasting accuracy. For most metrics, the proposed model outperforms existing methods in two of the data sets, while the best empirical performance in the other two data sets is attained by existing methods based on functional transformations of the densities.

MSC:
62G05 Nonparametric estimation
62G20 Asymptotic properties of nonparametric inference
62M10 Time series, auto-correlation, regression, etc. in statistics (GARCH)

Keywords:
random densities; Wasserstein metric; time series; distributional forecasting

Full Text: DOI