On the best uniform polynomial approximation to the checkmark function. (English)

Summary: The best uniform polynomial approximation of the checkmark function $f(x) = |x - \alpha|$ is considered, as $\alpha$ varies in $(-1, 1)$. For each fixed degree $n$, the minimax error $E_n(\alpha)$ is shown to be piecewise analytic in $\alpha$. In addition, $E_n(\alpha)$ is shown to feature $n-1$ piecewise linear decreasing/increasing sections, called V-shapes. The points of the alternation set are proven to be piecewise analytic and monotone increasing in $\alpha$ and their dynamics are completely characterized. We also prove a conjecture of Shekhtman that for odd $n$, $E_n(\alpha)$ has a local maximum at $\alpha = 0$.

MSC:
41-XX Approximations and expansions
42-XX Harmonic analysis on Euclidean spaces
41A10 Approximation by polynomials
41A50 Best approximation, Chebyshev systems

Full Text: DOI

References:
[1] Bernstein, S. N., Sur le meilleur approximation de $|x|$ par des polynomes de degrés donnés, Acta Math., 37, 1-57 (1913) · Zbl 44.0475.01

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