Güneysu, Batu


Summary: Let \((X,d,m)\) be an RCD*(K,N) space for some \(K \in \mathbb{R}, N \in [1, \infty)\), and let \(H\) be the self-adjoint Laplacian induced by the underlying Cheeger form. Given \(\alpha \in [0,1]\), we introduce the \(\alpha\)-Kato class of potentials on \((X,d,m)\), and given a potential \(V : X \to \mathbb{R}\) in this class, we denote with \(H_V\) the natural self-adjoint realization of the Schrödinger operator \(H + V\) in \(L^2(X,m)\). We use Brownian coupling methods and perturbation theory to prove that for all \(t > 0\), there exists an explicitly given constant \(A(V,K,\alpha,t) < \infty\), such that for all \(\Psi \in L^\infty(X,m), x, y \in X\) one has

\[
|e^{-tH_V}\Psi(x) - e^{-tH_V}\Psi(y)| \leq A(V,K,\alpha,t)\|\Psi\|_{L^\infty}d(x,y)^\alpha.
\]

In particular, all \(L^\infty\)-eigenfunctions of \(H_V\) are globally \(\alpha\)-Hölder continuous. This result applies to multi-particle Schrödinger semigroups and, by the explicitness of the Hölder constants, sheds some light into the geometry of such operators.

MSC:

81-XX Quantum theory
31-XX Potential theory

Full Text: DOI