A novel method to impose boundary conditions for higher-order partial differential equations. (English) Zbl 07487666

Summary: Partial differential equations (PDEs) with spatial derivatives of order higher than two are receiving increasing attention, partially due to the current popularity of the phase-field method. In the finite element community, the growth of Isogeometric Analysis and other discretization schemes that employ approximation spaces with high-order continuity has fostered the use of variational formulations that avoid the use of auxiliary unknowns representing derivatives of the solution. However, one of the caveats of this approach is the accurate and efficient imposition of boundary conditions, especially on complex geometries. This paper proposes a new method to impose boundary conditions naturally in the weak form of higher-order PDEs. Our method is based on a specially-designed weak form in which the boundary conditions to be imposed are weighted by derivatives of the weight functions. This requires multiple integrations by parts which are allowed due to the smoothness of the basis functions. The Cahn-Hilliard equation and the isothermal Navier-Stokes-Norteweg equations are used as examples to demonstrate the proposed method. We show that if the solutions of the PDEs are smooth enough, the proposed variational equations and the original PDEs are equivalent. We discretize the variational equations using Isogeometric Analysis. Convergence results of the proposed method agree with the best approximation errors of the basis functions. Numerical examples illustrate the applicability of the approach to mapped geometries with non-conformal grids.

MSC:
65-XX Numerical analysis
35-XX Partial differential equations

Keywords:
higher-order PDEs; contact angle boundary condition; phase-field; Cahn-Hilliard; Navier-Stokes-Korteweg

Full Text: DOI

References: