Hagelstein, Paul; Stokolos, Alex
Sharp weak type estimates for a family of Zygmund bases. (English) Zbl 07487895

Summary: Let $B$ be the collection of rectangular parallelepipeds in $\mathbb{R}^3$ whose sides are parallel to the coordinate axes and such that $B$ consists of parallelepipeds with side lengths of the form $s, 2^j s, t$, where $s, t > 0$ and $j$ lies in a nonempty subset $S$ of the integers. In this paper, we prove the following:

If $S$ is a finite set, then the associated geometric maximal operator $M_B$ satisfies the weak type estimate

$$\left| \{x \in \mathbb{R}^3 : M_B f(x) > \alpha \} \right| \leq C \int_{\mathbb{R}^3} \frac{|f|}{\alpha} \left(1 + \log^+ \frac{|f|}{\alpha}\right)$$

but does not satisfy an estimate of the form

$$\left| \{x \in \mathbb{R}^3 : M_B f(x) > \alpha \} \right| \leq C \int_{\mathbb{R}^3} \phi \left(\frac{|f|}{\alpha}\right)$$

for any convex increasing function $\phi : [0, \infty) \to [0, \infty)$ satisfying the condition

$$\lim_{x \to \infty} \frac{\phi(x)}{x(\log(1 + x))} = 0.$$

On the other hand, if $S$ is an infinite set, then the associated geometric maximal operator $M_B$ satisfies the weak type estimate

$$\left| \{x \in \mathbb{R}^3 : M_B f(x) > \alpha \} \right| \leq C \int_{\mathbb{R}^3} \frac{|f|}{\alpha} \left(1 + \log^+ \frac{|f|}{\alpha}\right)^2$$

but does not satisfy an estimate of the form

$$\left| \{x \in \mathbb{R}^3 : M_B f(x) > \alpha \} \right| \leq C \int_{\mathbb{R}^3} \phi \left(\frac{|f|}{\alpha}\right)$$

for any convex increasing function $\phi : [0, \infty) \to [0, \infty)$ satisfying the condition

$$\lim_{x \to \infty} \frac{\phi(x)}{x(\log(1 + x))^2} = 0.$$

MSC:

42B25 Maximal functions, Littlewood-Paley theory

Keywords:
maximal functions; differentiation basis

Full Text: DOI

References:


