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Non-spherical Harish-Chandra Fourier transforms on real reductive groups. (English)

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Summary: The Harish-Chandra Fourier transform, $f \mapsto Hf$, is a linear topological algebra isomorphism of the spherical (Schwartz) convolution algebra $C^p(G//K)$ (where $K$ is a maximal compact subgroup of any arbitrarily chosen group $G$ in the Harish-Chandra class and $0 < p \leq 2$) onto the (Schwartz) multiplication algebra $\mathcal{Z}(\mathfrak{g}^*)$ (of $n$-invariant members of $\mathcal{Z}(\mathfrak{g}^*)$, with $\epsilon = (2/p) - 1$). This is the well-known Trombi-Varadarajan theorem for spherical functions on the real reductive group, $G$. Even though $C^p(G//K)$ is a closed subalgebra of $C^p(G)$, a similar theorem has not however been successfully proved for the full Schwartz convolution algebra $C^p(G)$ except; for $C^p(G//K)$ (whose method is essentially that of Trombi-Varadarajan, as shown by M. Eguchi); for few specific examples of groups (notably $G = SL(2, \mathbb{R})$) and; for some notable values of $p$ (with restrictions on $G$ and/or on members of $C^p(G)$). In this paper, we construct an appropriate image of the Harish-Chandra Fourier transform for the full Schwartz convolution algebra $C^p(G)$, without any restriction on any of $G$, $p$ and members of $C^p(G)$. Our proof, that the Harish-Chandra Fourier transform, $f \mapsto Hf$, is a linear topological algebra isomorphism on $C^p(G)$, equally shows that its image $C^p(\hat{G})$ can be nicely decomposed, that the full invariant harmonic analysis is available and implies that the definition of the Harish-Chandra Fourier transform may now be extended to include all $p$-tempered distributions on $G$ and to the zero-Schwartz spaces.

MSC:

43A85 Harmonic analysis on homogeneous spaces
22E30 Analysis on real and complex Lie groups
22E46 Semisimple Lie groups and their representations

Keywords:

Fourier transform; reductive groups; Harish-Chandra’s Schwartz algebras

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References:

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