Carvajal-Rojas, Javier
Finite torsors over strongly $F$-regular singularities. (English) [Zbl 07491450]
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Summary: We investigate finite torsors over big opens of spectra of strongly $F$-regular germs that do not extend to torsors over the whole spectrum. Let $(R, m, k, K)$ be a strongly $F$-regular $k$-germ where $k$ is an algebraically closed field of characteristic $p > 0$. We prove the existence of a finite local cover $R \subset R^*$ so that $R^*$ is a strongly $F$-regular $k$-germ and: for all finite algebraic groups $G/k$ with solvable neutral component, every $G$-torsor over a big open of Spec $R^*$ extends to a $G$-torsor everywhere. To achieve this, we obtain a generalized transformation rule for the $F$-signature under finite local extensions. Such formula is used to show that the torsion of $\text{Cl} R$ is bounded by $1/s(R)$. By taking cones, we conclude that the Picard group of globally $F$-regular varieties is torsion-free. Likewise, this shows that canonical covers of $\mathbb{Q}$-Gorenstein strongly $F$-regular singularities are strongly $F$-regular.

MSC:
- 14B05 Singularities in algebraic geometry
- 14L15 Group schemes
- 13A35 Characteristic $p$ methods (Frobenius endomorphism) and reduction to characteristic $p$; tight closure
- 13A50 Actions of groups on commutative rings; invariant theory
- 13B05 Galois theory and commutative ring extensions
- 14F35 Homotopy theory and fundamental groups in algebraic geometry
- 14L30 Group actions on varieties or schemes (quotients)
- 16T05 Hopf algebras and their applications

Keywords:
$F$-regularity; $F$-signature; finite torsors; local Nori fundamental group-scheme

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