Empirical measure and small noise asymptotics under large deviation scaling for interacting diffusions. (English) J. Theor. Probab. 35, No. 1, 295-349 (2022)

Summary: Consider a collection of particles whose state evolution is described through a system of interacting diffusions in which each particle is driven by an independent individual source of noise and also by a small amount of noise that is common to all particles. The interaction between the particles is due to the common noise and also through the drift and diffusion coefficients that depend on the state empirical measure. We study large deviation behavior of the empirical measure process which is governed by two types of scaling, one corresponding to mean field asymptotics and the other to the Freidlin-Wentzell small noise asymptotics. Different levels of intensity of the small common noise lead to different types of large deviation behavior, and we provide a precise characterization of the various regimes. The rate functions can be interpreted as the value functions of certain stochastic control problems in which there are two types of controls; one of the controls is random and nonanticipative and arises from the aggregated contributions of the individual Brownian noises, whereas the second control is nonrandom and corresponds to the small common Brownian noise that impacts all particles. We also study large deviation behavior of interacting particle systems approximating various types of Feynman-Kac functionals. Proofs are based on stochastic control representations for exponential functionals of Brownian motions and on uniqueness results for weak solutions of stochastic differential equations associated with controlled nonlinear Markov processes.

MSC:

60F10 Large deviations
60K35 Interacting random processes; statistical mechanics type models; percolation theory
60B10 Convergence of probability measures
60H10 Stochastic ordinary differential equations (aspects of stochastic analysis)
93E20 Optimal stochastic control

Keywords:
large deviation principle; weakly interacting diffusions; mean field systems with common noise; Feynman-Kac functionals; controlled nonlinear Markov processes; mean field stochastic control problems; controlled McKean-Vlasov equations; Freidlin-Wentzell asymptotics

References:


Dawson, DA; Gärtner, J., Large deviations from the McKean-Vlasov limit for weakly interacting diffusions, Stochastics, 20, 4, 247-308 (1987) · Zbl 0613.60021 · doi:10.1080/17442508708833446


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