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**Sphere tangencies, line incidences and Lie’s line-sphere correspondence.**  
(English)  

Summary: Two spheres with centers $p$ and $q$ and signed radii $r$ and $s$ are said to be in contact if 
$$
|p - q|^2 = (r - s)^2.
$$
Using Lie’s line-sphere correspondence, we show that if $F$ is a field in which -1 is not a square, then there is an isomorphism between the set of spheres in $F^3$ and the set of lines in a suitably constructed Heisenberg group that is embedded in $(F[i])^3$; under this isomorphism, contact between spheres translates to incidences between lines. In the past decade there has been significant progress in understanding the incidence geometry of lines in three space. The contact-incidence isomorphism allows us to translate statements about the incidence geometry of lines into statements about the contact geometry of spheres. This leads to new bounds for Erdős’ repeated distances problem in $F^3$, and improved bounds for the number of point-sphere incidences in three dimensions. These new bounds are sharp for certain ranges of parameters.

**MSC:**

52C35  

**Full Text:** DOI

**References:**


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