Summary: Let $K$ be a convex polyhedron and $\mathcal{F}$ its Wulff energy, and let $\mathcal{C}(K)$ denote the set of convex polyhedra close to $K$ whose faces are parallel to those of $K$. We show that, for sufficiently small $\varepsilon$, all $\varepsilon$-minimizers belong to $\mathcal{C}(K)$.

As consequence of this result we obtain the following sharp stability inequality for crystalline norms: There exist $\gamma = \gamma(K, n) > 0$ and $\sigma = \sigma(K, n) > 0$ such that, whenever $|E| = |K|$ and $|E \Delta K| \leq \sigma$, then

$$\mathcal{F}(E) - \mathcal{F}(K^*) \geq \gamma |E \Delta K^*| \leq \sigma$$

for some $K^* \in \mathcal{C}(K)$.

In other words, the Wulff energy $\mathcal{F}$ grows very fast (with power 1) away from $\mathcal{C}(K)$. The set $K^* \in \mathcal{C}(K)$ appearing in the formula above can be informally thought as a sort of “projection” of $E$ onto $\mathcal{C}(K)$.

Another collar of our result is a very strong rigidity result for crystals: For crystalline surface tensions, minimizers of $\mathcal{F}(E) + \int_E g$ with small mass are polyhedra with sides parallel to the those of $K$. In other words, for small mass, the potential energy cannot destroy the crystalline structure of minimizers. This extends to arbitrary dimensions a two-dimensional result obtained in [9].

MSC:

26-XX Real functions
60-XX Probability theory and stochastic processes

Full Text: Link