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Deformation of rational singularities and Hodge structure. (English) Zbl 07550578
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Summary: For a one-parameter degeneration of reduced compact complex analytic spaces of dimension $n$, we prove the invariance of the frontier Hodge numbers $h^{p,q}$ (that is, those with $pq(n-p)(n-q) = 0$) for the intersection cohomology of the fibers and also for the cohomology of their desingularizations, assuming that the central fiber is reduced, projective, and has only rational singularities. This can be shown to be equivalent to the invariance of the dimension of the cohomology of the structure sheaf since we can prove the Hodge symmetry for all the Hodge numbers $h^{p,q}$ together with $E_1$-degeneration of the Hodge-to-de Rham spectral sequence for nearby fibers, assuming only the projectivity of the central fiber.

For the proof of the main theorem, we calculate the graded pieces of the induced $V$-filtration for the first non-zero member of the Hodge filtration on the intersection complex Hodge module of the total space, which coincides with the direct image of the dualizing sheaf of a desingularization. This calculation also implies that the order of nilpotence of the local monodromy is smaller than in the general singularity case by 2 in the situation of the main theorem assuming further smoothness of general fibers.

MSC:
14D07 Variation of Hodge structures (algebro-geometric aspects)
14B07 Deformations of singularities
32S30 Deformations of complex singularities; vanishing cycles

Keywords:
rational singularities; limit mixed Hodge structure; frontier Hodge numbers

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