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An abstract approach to model theory. (English) [Zbl 0788.47005]


In these notes we shall summarize our recent results on the uses of hereditary polynomials in operator theory. In order to map out the theory in an economical manner we shall purposely omit proofs of all but the most trivial facts and adopt an expository and at times informal style.

The type of structure that hereditary polynomials are adapted to analyze is that of a lifting theorem. If $\mathcal{B}$ and $\mathcal{F}$ are collections of operators with $\mathcal{B} \subseteq \mathcal{F}$ and $\mathcal{B}$ closed with respect to direct sums and unital representations we say that a theorem is a lifting theorem if it has the form

0.1 If $T \in \mathcal{F}$ then there exists $B \in \mathcal{B}$ and $\mathcal{N}$ such that $\mathcal{N}$ is an invariant subspace for $B$ and $T = B|\mathcal{N}$.

One thinks of $\mathcal{B}$ as being a highly distinguished subcollection of $\mathcal{F}$ whose elements possess a highly developed model. The long-term goal in the case when 0.1 holds is to use the model for elements of $\mathcal{B}$ to study the elements of $\mathcal{F}$. Two pre-eminent examples are the theory of subnormal operators ($\mathcal{B} = \text{normals}$, $\mathcal{F} = \text{subnormals}$), and the Nagy-Foias, deBranges-Rovnyak theories of contractions ($\mathcal{B} = \text{coisometries}$, $\mathcal{F} = \text{contractions}$).

For the entire collection see [Zbl 0784.00016].

MSC:

- 47A45 Canonical models for contractions and nonselfadjoint linear operators
- 47A60 Functional calculus for linear operators
- 47B20 Subnormal operators, hyponormal operators, etc.
- 47A66 Quasitriangular and nonquasitriangular, quasidiagonal and nonquasidiagonal linear operators

Keywords:
hereditary polynomials in operator theory; lifting theorem; invariant subspace; subnormal operators